

An Extensible Approach for Modeling the Impact of Deterministic and Stochastic External Inputs in the Power Flow

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Abstract—This paper presents an approach for modeling the impact of weather and other external inputs (EXIs) in the power flow or optimal power flow (OPF) with consideration of both deterministic and stochastic models. The need for this work is due to growing dependence of large-scale grids on the weather and the potential need to model the impact of many other EXIs. The work presents a relatively easy to implement and extensible modeling approach for directly including EXIs in the power flow or OPF by extending the capability of contingency analysis to do Monte Carlo simulations. Results are demonstrated using earthquake scenarios on two synthetic grids with 37 and 993 buses, respectively.

Index Terms—power flow, Monte Carlo simulation, contingency analysis, earthquakes, visualization

I. INTRODUCTION

“If you fail to plan, you are planning to fail” is a quote widely attributed to Benjamin Franklin that certainly applies to large-scale electric grids. Of course, the electric grid is planned, and its generally excellent overall performance in the past is certainly a testament, in part, to the skill of the many people involved in this task. However, the grid is rapidly changing, as is the world itself with natural hazards becoming increasingly costly. The relatively easily controlled, fossil fuel-based grid of the past is rapidly being replaced by one with much more weather-dependent renewable generation, and the load is also changing including potentially large new demands from data centers, and the further electrification of various industries. At the same time extreme weather, and other potentially severe resiliency events, which will collectively be referred to here as External Inputs (EXIs), are making planning and operating the grid much more challenging. Hence a germane change to Franklin’s quote would be, “If you fail to plan effectively, you are planning to fail.” To meet these challenges, the tools power system planners use must be enhanced. The purpose of this paper is to present an easy to implement and extensible method for modeling both deterministic and stochastic EXIs in the power flow and optimal power flow (OPF) by extending the capabilities of contingency analysis (CA) to do Monte Carlo simulations.

The North American Electric Reliability Corporation (NERC) defines transmission planning as, “Documented evaluation of future transmission system performance and

corrective action plans to remedy identified deficiencies” [1]. Planning can occur over timeframes ranging from decades down to almost real-time [2]. While there have always been planning challenges, it had been easier in the past when generation and transmission were planned together, and the load growth was relatively predictable [3]. In many locations worldwide this has now changed dramatically, with generation often developed decoupled from transmission planning using an interconnection queue process [4]. In addition, the grid has become much more weather dependent, with large amounts of solar and wind generation now common in many locations.

Also, given the criticality for reliable electricity to society there is a need to enhance the grid’s resilience to a variety of EXIs [5]. The list of external events that could be considered is potentially large with Figure 3.1 in [5] listing thirteen different categories, ranging from those with a very short amount of warning time such as earthquakes and physical attacks, to those developing over days or weeks including hurricanes, volcanic events and droughts. Many of these events are naturally occurring, while some are human-caused. While this paper presents a general and extensible approach for modeling almost all EXIs, its specific focus is on earthquakes.

The remainder of the paper is organized as follows. The next section provides further background and brief literature review. The following section then presents the approach demonstrating it on a 37-bus power system case. Section IV then provides a larger case example with a 993-bus grid. The final section contains a conclusion and directions for future research. All the results and visualizations in the paper are done using solution approach implemented in PowerWorld Simulator Version 25.

II. BACKGROUND

While there are many tools used with power system planning, two general categories are 1) those that model the steady-state operation of the grid, and 2) those that model the grid dynamics during disturbances. The focus of this paper is on steady-state. Common tools used for steady-state analysis are 1) the power flow (or load flow), which assumes an electric grid is operating at a constant frequency, 2) the optimal power flow (OPF), which extends the power flow by varying controls, such as generator real power outputs, to minimize a cost function subject to a number of constraints such as transmission line or transformer limits, and 3) contingency analysis, which sequentially applies a set of statistically likely contingencies to either the power flow or OPF results. Commercial software packages usually have extensive feature sets for all of these tools.

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Given the potential impacts EXIs can have on the grid, over the years there have been a number of papers on externally modelling the impacts of EXIs in the power flow. That is, the EXIs are modeled in a separate software package that is used to produce output that can be used to modify the power flow case. Which EXIs should be included is problem dependent. However, some are much more widely applicable than others. For example, with the large amount of solar and wind generation in many grids, in which their outputs are EXI dependent, at least some weather information (e.g., wind speed, solar irradiance, temperature) is used to adjust their outputs. To deal with other weather-related events, such as hurricanes, ice storms, or derechos, this information could be expanded to include wind gusts and precipitation. Other events such as earthquakes, wildfires, geomagnetic disturbances (GMDs) or volcanic events would require additional EXI datasets. Examples of prior work in this area include [6], [7], [8], [9]. The general need for better weather modeling in power system planning is the focus of [10], whereas [11] and [12] show how weather information can be directly incorporated into power flow models.

As noted in [12], the use of EXIs in the power flow requires addressing three main issues. First, the power flow case must have geographic information for its components. While this had not been the case previously, nowadays power flow cases either already have the geographic location of the grid components, usually grouped by substation, or it can be readily found. Also, for research purposes there are now also a wide variety of large-scale synthetic electric grid models with geographic information [13], [14]; a public repository for such cases is described in [15].

Second, the EXI information needs to be available over the footprint for the power flow case. For example, [16] contains public domain information on earthquake risk, with Figure 1 showing with the data for the ten-percent in 50 year probability of peak ground acceleration, expressed as a percentage of gravity (PGAg) for the contiguous US (CONUS). While the input is from [16], this figure shows the 611,309 data values (expressed on a 0.05-degree grid) contained in the power flow models used later in this paper. Then to use this EXI information in the power flow requires that it be available at the location of the grid components, such as transformers for earthquake risk. How much spatial resolution is required in the EXI dataset depends on the application, with tradeoffs between accuracy, complexity and dataset size. Regardless, EXIs are seldom available exactly at the power grid component location, so some sort of 2D data interpolation is required. While there is no best method for all situations, in this paper bilinear interpolation is used when the data is available for a grid (i.e., Figure 1) and Delaunay triangulation otherwise.

The third issue associated with using EXIs in the power flow, and getting to the main purpose of this paper, is to relate the EXI to a power flow value. Here the approach builds on that of [12], which in turn is motivated by what is used with power system stability code. That is, the development of a potentially large number of model types to represent the

impact of different EVIs on different power system devices, with parameters used to customize the models. Like the stability model approach, this is a straightforward and easily extensible approach. Here these are referred to as PFW models, a term that originally was “power flow weather” but now has been generalized to “power flow whatever.”

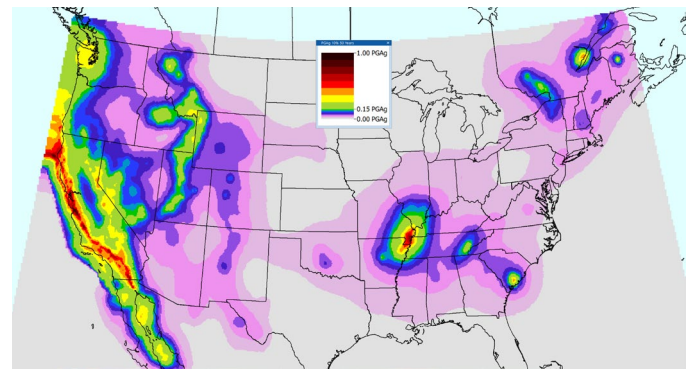


Figure 1: CONUS Ten-percent probability of exceedance in 50 years PGAg

These PFW models can be divided into two main classes: deterministic and stochastic. As an example, Figure 2 shows four deterministic PFW models for wind turbine generators in which a piecewise linear curve is used to relate the hub height wind speed to the normalized maximum power output with most of the data from [6]. An example of a stochastic PFW is shown in Figure 3, with a piecewise linear curve now used to relate the PGAg experience at a transformer to its probability of failure. A useful reference on grid component fragility curves, and the source of the data for Figure 3 is given in [17]. The next section presents an approach for modeling the PFWs with an emphasis on the stochastic models.

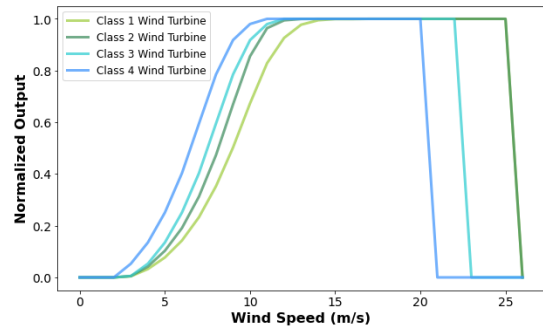


Figure 2: Piecewise Linear Curves for Four Wind Turbine Generators PFWs

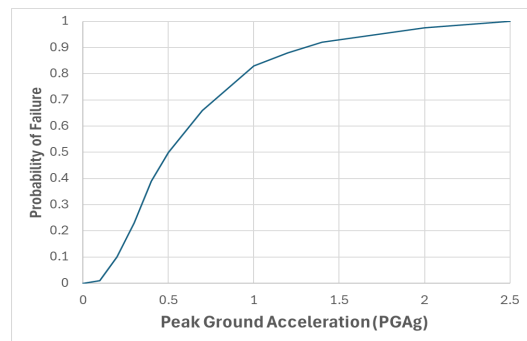


Figure 3: Generic Transformer Earthquake Fragility Curve

III. PFW STOCHASTIC MODELING APPROACH

As has been described previously [12] handling deterministic PFWs is relatively straightforward, with the PFW models used to modify the power system parameters before the power flow or OPF is solved. This section presents an approach for the stochastic PFWs, in which there is now uncertainty in the power flow parameters. For example, if a transformer is modeled using the fragility curve in Figure 3, whether it is assumed to have failed due to an earthquake is expressed as a probability, resulting in uncertainty with its status parameter.

Dealing with uncertainty in the power flow input parameters has been a consideration throughout the history of digital power flows. A standard, and quite useful approach that has been used for simulations in general for more than 70 years is the Monte Carlo method [18] in which some of the input parameters are assumed to behave as random variables, and then a number of repeated deterministic power flows are solved. Statistical techniques are then used to summarize the results. An alternative is the use of the analytic stochastic power flow [19], [20]. Often Monte Carlo simulations are used to validate the results from the analytic approach, and sometimes the two methods are combined [21].

In theory either of these approaches could be used to implement stochastic PFW models. However, as noted in [21] advantages of the Monte Carlo approach include conceptual simplicity (e.g., each scenario is seen as a possible operating condition) and flexibility to incorporate more complex modeling features. Both of these are characteristics that are needed when implementing the extensible stochastic PFW model approach presented here. In addition, when applying PFWs to model severe EXIs often the power flow does not converge. The issue then is how easily extend the existing, and often quite complex, deterministic power flow tools to accommodate stochastic models.

The solution presented here is to leverage the functionality provided in the contingency analysis (CA) application. CA, which dates back about 50 years [22], sequentially considers a set of statistically likely events (contingencies, such as a branch or generator outage), solving a deterministic power flow for each. However, over the years the CA has been extended so that modern commercial CA tools include much more than simple element outages. They now embed logic for relay actions, manual operator interventions, and automated Remedial Action Schemes (RAS) that conditionally apply actions or dynamically determine generator tripping levels. Modeling of generator voltage control, such as line drop and reactive current compensation, is also integrated. Additionally, tools can dynamically determine which breakers must open to isolate a device, allowing for the accurate modeling of complex substation topologies even during breaker or disconnect maintenance. Because a single initiating contingency can trigger dozens of subsequent automated actions, these tools must and do provide detailed event logs of what occurred to help engineers navigate the resulting complexity. CA also can have extensive functionality for showing and visualizing not only what happened during an

individual contingency, but also for summarizing the full CA results. Also, the CA, like the Monte Carlo method is inherently parallelizable, meaning the most commercial tools already support parallel solutions. These characteristics are exactly the functionality needed when dealing with the impact of the stochastic PFW models, which could represent quite extreme operating conditions.

The approach implemented here for using CA to do Monte Carlo analysis required for stochastic PFW models is as follows. First, solve the power flow or OPF based upon the desired operating point taking into account as needed any deterministic PFWs. Second, use the CA tool to generate a set of empty contingencies that will be used for the Monte Carlo analysis. How many depends on the desired confidence, with the Law of Large Numbers and Central Limit Theorem providing insight [23], with good results obtained with a relatively modest number of contingencies. In the examples presented here 500 contingencies are used. For each a set of random numbers is generated corresponding to the number of stochastic PFW models. The contingency is then solved using the standard process, and all the capabilities of the CA tool used to help interpret the results.

An initial example of this procedure applied to modeling earthquake risk is demonstrated using the 37-bus electric grid from [24], whose online is shown in Figure 1 and is publicly available at [25]. Overall, this case models a grid with about 800 MW of load using a 345/138/69 kV transmission system with 37 buses grouped in 27 substations, 9 generators, 43 transmission lines, and 14 transformers. While the case itself does not have geographic coordinates, assume that it is located in a region in which the 50-year, ten-percent likelihood of exceedance PGAg value is 0.5 (a value exceeded by roughly the red region in Figure 1), with each of the transformers modeled with the fragility curve in Figure 3. For a PGAg of 0.5 the probability of failure for each transformer is about 50%. Hence this corresponds to a very severe event.

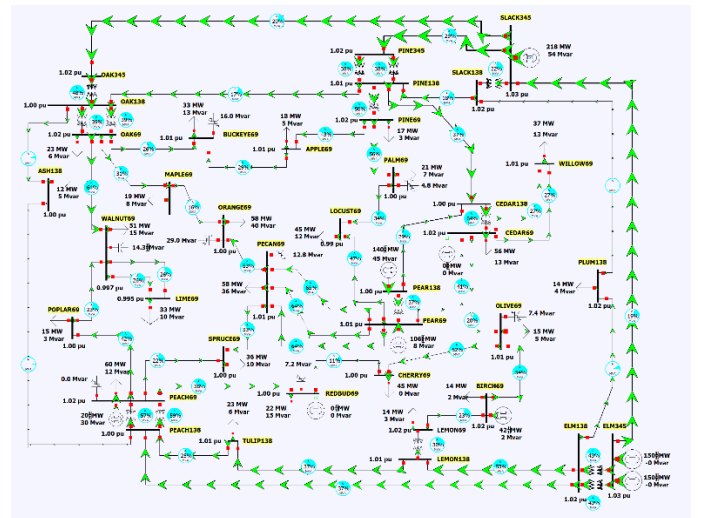


Figure 4: 37-Bus System Online

The likelihood of this grid to experience problems during the earthquake scenario is then evaluated using 500

contingencies. Some of the results are shown in Figure 5 and the oneline for the first stochastic contingency is shown in Figure 6. Of course, usually one would not be interested in the particulars of a single contingency, but if desired this information is available. Of most interest would be the statistical properties for the entire CA results, discussed next.

	Data View	Solved	Total Stochastic Models with Open Devices	Violations	Max Branch %	Min Volt
1	Stochastic_1	YES	6	7	175.280	0.949
2	Stochastic_2	YES	7	3	168.089	
3	Stochastic_3	NO	10	Unsolved		
4	Stochastic_4	YES	7	2	142.445	
5	Stochastic_5	YES	7	37	219.321	0.775
6	Stochastic_6	YES	6	4	120.078	0.943
7	Stochastic_7	YES	7	23	153.842	0.906
8	Stochastic_8	NO	8	Unsolved		
9	Stochastic_9	YES	8	3	140.347	
10	Stochastic_10	YES	7	4	142.694	0.945
11	Stochastic_11	YES	6	3	143.163	0.948
12	Stochastic_12	YES	5	3	129.451	
13	Stochastic_13	YES	8	18	177.515	0.911
14	Stochastic_14	YES	6	6	146.884	0.946
15	Stochastic_15	YES	8	1	110.267	
16	Stochastic_16	NO	9	Unsolved		
17	Stochastic_17	NO	10	Unsolved		
18	Stochastic_18	YES	6	4	136.410	
19	Stochastic_19	YES	7	2	131.921	
20	Stochastic_20	NO	8	Unsolved		
21	Stochastic_21	YES	7	6	116.752	
22	Stochastic_22	YES	8	4	135.280	
23	Stochastic_23	YES	8	9	143.271	0.933
24	Stochastic_24	YES	6	1	115.960	
25	Stochastic_25	YES	6	3	126.476	
26	Stochastic_26	YES	6	2	132.950	
27	Stochastic_27	YES	4	3	126.521	
28	Stochastic_28	YES	5	2	108.234	
29	Stochastic_29	YES	4	2	107.741	

Figure 5: 37-Bus Example Stochastic Contingency Results

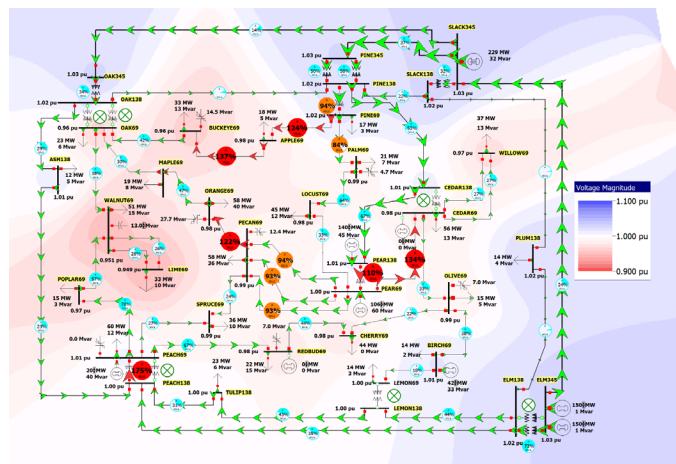


Figure 6: 37-Bus Stochastic_1 Case Details

In assessing the earthquake resiliency of this grid, the most important statistic is the number of failed transformers, with Figure 7 showing a histogram of the distribution. Given that each transformer has a 50% chance of failure for the specified value of PGAg, this is close to the expected result Gaussian distribution with an expected value of half the 14 transformers failing. Another important statistic from a resiliency point of view is that for 123 contingencies (24.6%) the power flow did not converge, indicating a likely system-wide collapse.

While these results are interesting in and of themselves, from a planning perspective a goal could be to use them to give insight into how to improve this grid's earthquake resiliency. For example, a design question might be, "if funds are available to strengthen just two transformers, which two should be chosen"? One approach would be to look at the transformers whose failures are most associated with power flow non-convergence. In considering the 123 non-convergent

contingencies the 345/138 kV transformer at the Oak substation (shown in the upper left on the oneline) is associated with the most contingencies (93), with one of the 345/138 kV transformer at Pine substation (upper middle) associated with the second highest number (91). In contrast, the Peach 138/69 kV only is associated with 69 non-convergent contingencies.

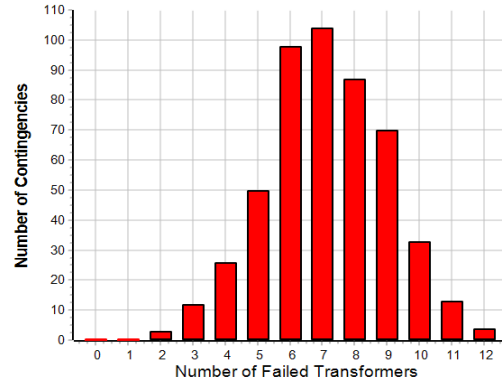


Figure 7: Histogram of Number of Failed Transformers

To consider the implications of strengthening these two transformers the results can be redone with their fragility curves modified. How the curves should be modified depends on the design solution. In the best-case scenario, their PFW models could just be disabled indicating they would no longer fail. A more likely scenario would be to shift the Figure 3 curve towards the lower left. In the PFW model used with this example (shown in Figure 8), the first three parameters make it easy to modify the fragility curve either by 1) scaling the curve along the x-axis with a value greater than one making the transformer more durable, 2) shifting the curve along the x-axis with a positive value making it more durable, or 3) shifting the curve along the y-axis with a positive value making it more fragile.

Parameter	Value
CurveXScale	1.0000
CurveYShift	0.00000
PGAg1	0.00000
PGAg2	0.1000
PGAg3	0.2000
PGAg4	0.3000
PGAg5	0.4000
PGAg6	0.5000
PGAg7	0.7000
PGAg8	1.0000
PGAg9	1.2000
PGAg10	1.4000
PGAg11	2.0000
PGAg12	2.5000
PGAg13	0.00000
PGAg14	0.00000
Prob1	0.00000
Prob2	0.01000
Prob3	0.1000
Prob4	0.2300
Prob5	0.3900
Prob6	0.5000
Prob7	0.6600
Prob8	0.8300
Prob9	0.8800
Prob10	0.9200
Prob11	0.9750
Prob12	1.0000
Prob13	0.00000
Prob14	0.0000
Prob15	0.0000
Prob16	0.0000
Prob17	0.0000
Prob18	0.0000
Prob19	0.0000

Figure 8: Transformer Earthquake PFW Model Data Entry Dialog

To facilitate comparing two results with modified PFW models in the implementation present here there is an option to determine whether the random variables associated with each contingency are updated. If they are updated, then the results would naturally vary some even if there are no case or PFW model changes. For example, rerunning the previous scenario without changing the PFW models but updating the random variables resulted in 116 non-convergent contingencies while doing it a third time resulted in 98. The alternative is to not update the random variables, resulting in the same outcome if nothing is changed, and a more informative comparison when the PFW models are changed. If the PFW models at previously mentioned transformers are disabled using the original random variables the number of non-convergent contingencies drops from 123 (24.6%) to 46 (9.2%), whereas if their fragility curves are just shifted to the right by 0.25 PGAg (with a new probability of failure at 0.5 PGAg reduced from 50% to 6.5%) there are 76 (15.2%) non-convergent contingencies.

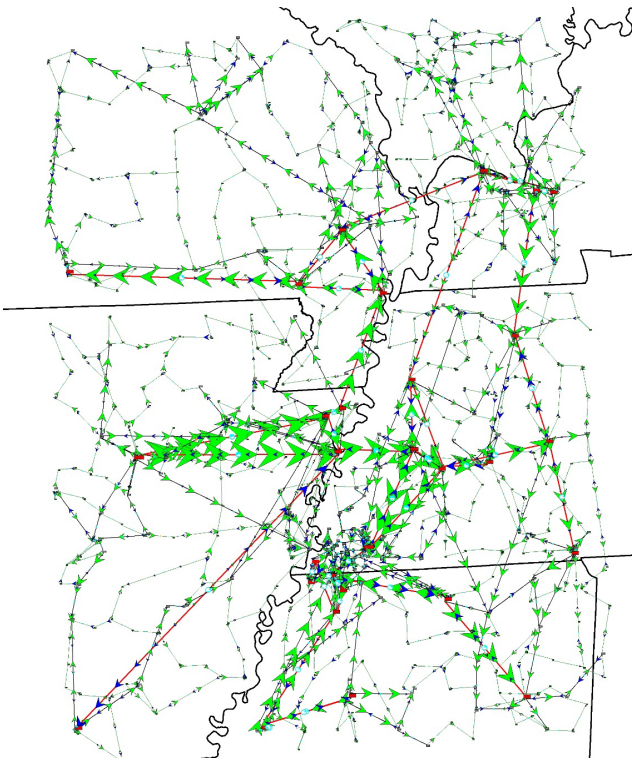


Figure 9: 993-Bus System Oneline

IV. LARGER CASE EXAMPLE

Here the approach is demonstrated on the 993-bus synthetic electric grid whose oneline is shown in Figure 7 with the full grid publicly available at [15]. In the figure the green arrows visualize the flow of real power with the arrow's size proportional to its real power flow; the line's background color indicates is nominal voltage with red for 345 kV and black for 138 kV or 69 kV. Overall, this case has 993 buses in 561 substations with 188 generators, 974 transmission lines, 407 transformers, and a total load of 11,200 MW. Its total

generation capacity is 19,600 MW of which about 1800 MW is solar; the solar power is modeled with PFW models in which its real power output is determined by the solar irradiance. The weather used is July 25, 2023 at 1800 UTC (1 pm local time) which had high irradiance over the region so the solar is at its maximum output. More germane here, the grid is located in the central portion of the US, covering parts of the states of Missouri, Illinois and Kentucky in its north, and Arkansas, Tennessee and Mississippi in its south; this places it right on top of the New Madrid seismic zone. Again, each transformer is modeled with the Figure 3 fragility curve.

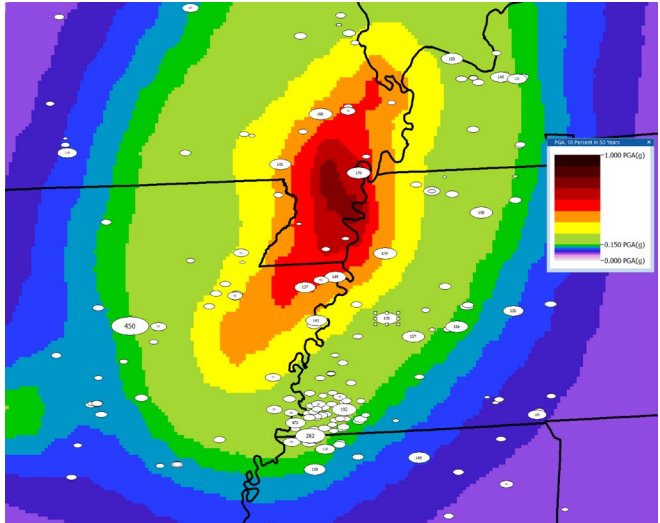


Figure 10: 993-Bus System Transformer GDV Display on PGAg Background

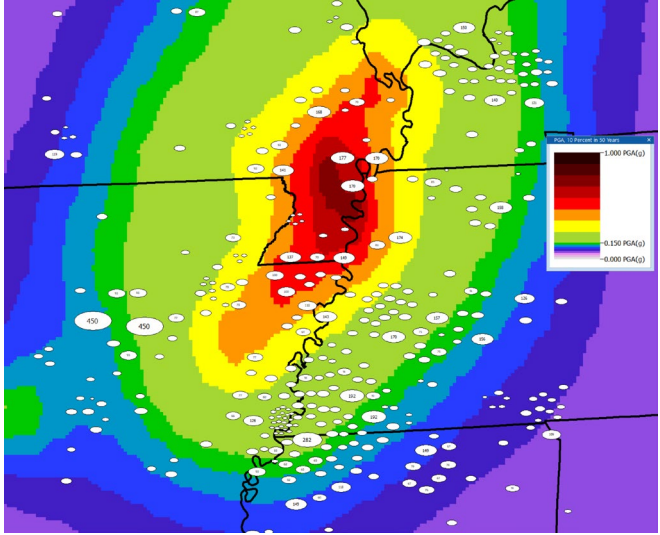


Figure 11: 993-Bus System Transformer GDV Display with Auto Layout

To better visualize the geographic location of the case's transformers relative to the PGAg values, Figure 10 uses the geographic data view (GDV) approach of [26] to use white oval's for each transformer with the size of the oval proportional to the real power flowing through the transformer; the text value in each oval shows this value in MW. However, since there are often multiple transformers at a single geographic location (i.e., a substation), Figure 11

repeats the Figure 10 GDVs except using the force-based layout algorithm of [27] to remove this overlap at the expense of geographic accuracy.

The process from the previous section can then be used to help quantify the earthquake resilience of this grid with now the actual Figure 1 PGAG values used for each transformer depending on its location. Again 500 contingencies are used, with the histogram for the number of failed transformers given in Figure 12. Similarly to the 37-bus case, there are a number of non-convergent contingencies 48 (9.6%), indicating an earthquake matching the Figure 1 values has at least some likelihood of causing a system-wide collapse, at least for this 993-bus synthetic grid. Also, four additional contingencies converged to likely alternative power flow solutions, detected using the fast-screening method of [28].

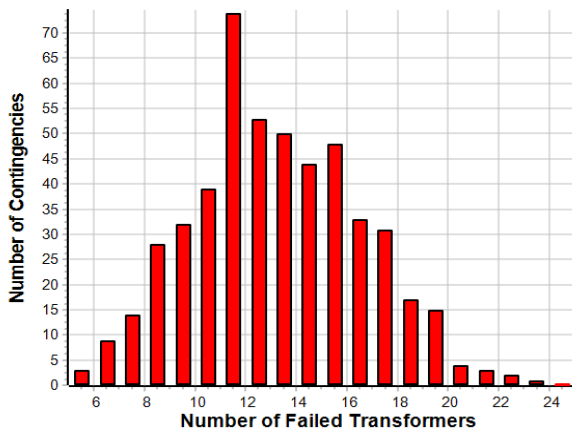


Figure 12: 993-Bus System Histogram of Failed Transformers

V. CONCLUSION AND FUTURE DIRECTIONS

This paper has shown how earlier work using the PFW approach for deterministic models can be extended to stochastic models by modifying the existing contingency analysis tool to do Monte Carlo simulations. The approach is demonstrated for earthquake events with example results given for 37-bus and 993-bus grids. There are many directions for future work. This includes taking advantage of the wide variety of results in the existing literature to expand the classes of EXIs considered and to greatly expand the number of PFW model classes. Also, for some EXIs with time frames on the order of seconds to minutes, the impact of the EXIs could also be considered in time-domain stability simulations.

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